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# 3D EIT imaging with Green's functions

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**Abstract:** A theoretical approach that uses Green functions in order to perform 3D EIT imaging of low-conductivity disturbances is briefly described. Applying the Green's function solution in an integral governing equation, a sensitivity matrix is constructed, from which the solution is obtained.

## 1 Introduction

Although 2D imaging is applied in the majority of EIT medical applications, there exist numerous effects associated with the examination domain's depth and volume. This creates the necessity to perform 3D imaging even if only a single plane's properties are of interest. Here, we propose an alternative approach that uses the Green function vector theorem.

## 2 Methods

### 2.1 The Green function

The fundamental mathematical background for the 2D case has been introduced in [1]. If $\boldsymbol{r} = (x, y, z)$ is the observation point, $\boldsymbol{r'} = (x', y', z')$ an internal "source" point and $\boldsymbol{r_+}$, $\boldsymbol{r_-}$ the current source electrode vectors (point-electrode model), then the EIT equation is:

$$\nabla\sigma\nabla V + \sigma\nabla^2 V = I[\delta(\boldsymbol{r} - \boldsymbol{r_+}) - \delta(\boldsymbol{r} - \boldsymbol{r_-})] \qquad (1)$$

If $\Omega \in R^3$ is the examination domain and assuming that no currents flow through the boundaries, application of the Green's integral theorem gives:

$$V(\boldsymbol{r}) = \iiint_\Omega G(\boldsymbol{r}, \boldsymbol{r'})\frac{\nabla\sigma(r')}{\sigma(r')}\nabla V(\boldsymbol{r'}, \boldsymbol{r_+}, \boldsymbol{r_-})d\Omega + V_{ref}(\boldsymbol{r}, \boldsymbol{r_+}, \boldsymbol{r_-}) \qquad (2)$$

The Green function $G(\boldsymbol{r}, \boldsymbol{r'})$ verifies the Poisson PDE $\nabla^2 G(\boldsymbol{r}, \boldsymbol{r'}) = -\delta(x - x')\delta(y - y')\delta(z - z')$ along with the non-current flow Neumann boundary condition, $\nabla V\boldsymbol{r} = 0$. The solution can be found either analytically for canonical or conformal geometries or numerically for more complex cases. In this work, we are interested in applications for maligant detection at female breast inserted in a cylindrical domain filled with conductive water; thus, we emphasize in a cylindrical geometry $\Omega_c$ with radius $R_o$. Multiple electrode planes are attached to the cylinder. The analytic solution on $\Omega_c$ includes a source term $G_o(\boldsymbol{r}, \boldsymbol{r'}) = \frac{1}{2\pi|\boldsymbol{r} - \boldsymbol{r'}|}$ and a correction term, approximated by the method of images: $G_1(\boldsymbol{r}, \boldsymbol{r'}) = \frac{1}{2\pi|\boldsymbol{r_{im}} - \boldsymbol{r'}|}$, where $\boldsymbol{r_{im}}$ is the symmetric of $\boldsymbol{r}$ with respect to $\partial\Omega_c$. [2]

### 2.2 System formulation and solution

Due to the singular behavior of $G$ on the electrodes we set two assumptions: Firstly, the electrodes are not placed at $\partial\Omega_c$, but in a small inner distance ($|R_o - \epsilon|$). Secondly, (2) is solved in an inner area of interest $A \subset \Omega_c$ which does not include the electrodes. Furthermore, a linearization $\nabla V \simeq \nabla V_{ref}$ is performed around the background conductivity $\sigma_{ref}$. The conductivities are expressed globally with an exponential-type base ($n$ even):

$$ln(\sigma(\boldsymbol{r'})) = \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{L} a_{ijk} e^{-\frac{[(x'-x_i)^n + (y'-y_j)^n + (z'-z_k)^n]^{\frac{1}{n}}}{D^2}} \qquad (3)$$

where $\boldsymbol{r_{ij}} = (x_i, y_j, z_k)$ is a central voxel point. $A$ is discretized in a $M \times N \times L$ voxel area and (2) is numerically solved, leading to an equation system $Sa = V_{ref}(\boldsymbol{r}) - V(\boldsymbol{r})$, $a = [a_{ij}]$. $S$ is a sensitivity matrix, with similar behavior to a Jacobian matrix. The system can be solved with state-of-the-art inverse problem approaches (Gen. Tikhonov reg., priors).

## 3 Results

Simulation tests were performed using MATLAB. The forward model was created with the NETGEN feature of EIDORS. A result is shown in Fig. 1 (NOSER prior):

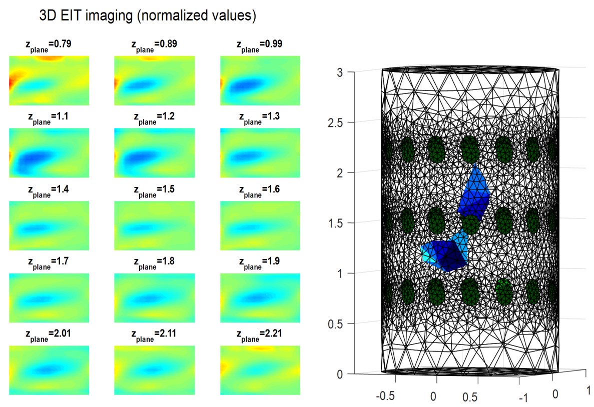


**Figure 1:** 3D image reconstruction for 2 small pertrurbations

## 4 Conclusions

In this paper, we proposed an algorithm that performs 3D imaging using Green's integral theorem. Results seem promising, with improvements such as extension for the complete electrode model and iterative approaches for more intense conductivity changes to be in the works.

## 5 Acknowledgements

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## References

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